# Pre-class Warm-up!!

A particle moves along a path $c(t) = (t^3 - 3, 2t^2 + 1)$

What is the speed of the particle at $t = 1$?

a. 3 units /sec

b. 4 units /sec

c. 5 units /sec ✓

d. √ 5 units /sec

e. None of the above.

$c'(t) = (3t^2, 4t)$

$c'(1) = (3, 4)$

Speed $= \| c'(1) \| = \sqrt{3^2 + 4^2} = 5$

## 4.1 Paths again. Acceleration and Newton's Second Law

We recall:

- a path is a mapping c : [a,b] -> R^n
- we can differentiate it to get a velocity vector v = c′(t)
- it satisfies some rules:
  sum rule, scalar multiplication rule, chain rule and NEW dot product rule, cross product rule
- Also new: acceleration vector. $a = v'(t) = c''(t)$
- Newton: force = mass x acceleration

Terminology I will not use: regular path.

A path $c$ is regular if $c'(t) \neq 0$ for all $t$.

Example: $c(t) = t^3$ parametrizes $\mathbb{R}$ and is not regular.

Typical HW questions:
Find the velocity and acceleration vectors

Verify the rules.

Given values of c′′(t), c′(0) and c(0) find c.

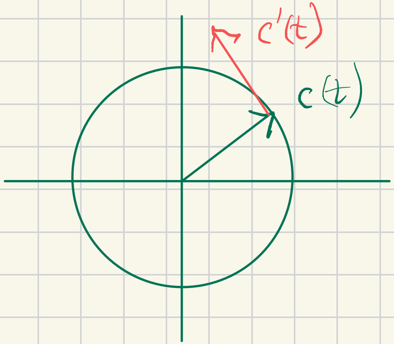
Find the force on a particle under some given acceleration.

Rules for doing $c'(t)$.

1. $(ac + bd)' = ac' + bd'$
2. chain rule
3. $\frac{d}{dt}(c \cdot d) = c' \cdot d + c \cdot d'$.
4. If $c, d : [a,b] \to \mathbb{R}^3$ then $(c \times d)' = c' \times d + c \times d'$

Examples:

1. (Like qn 20) If $\| c(t) \|$ is constant then $c'(t)$ is perpendicular to $c(t)$ for all $t$.




Solution: If $\|c(t)\|$ is constant so is $c(t)\cdot c(t) = \|c(t)\|^2$

$$\frac{d}{dt}\big(c(t)\cdot c(t)\big) = c'(t)\cdot c(t) + c(t)\cdot c'(t)$$

$$= 2c'(t)\cdot c(t) = 0$$

Thus $c'(t)$ is perpendicular to $c(t)$.

$\square$

Example: find the force on a particle in circular motion, of mass 1, tracing a path $R(t) = (\cos t, \sin t)$

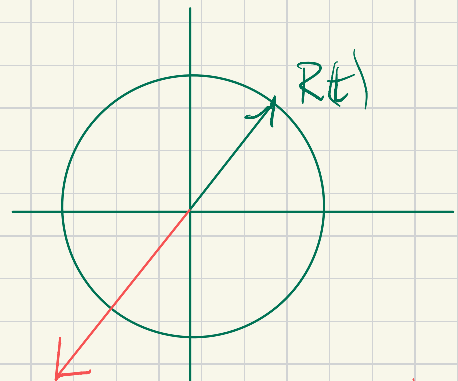
Change to mass $= 3$, $R(t) = (\cos 2t, \sin 2t)$

Solution:

Velocity is $v = R'(t) = (-2\sin 2t, 2\cos 2t)$

Acceleration is $a = v' = (-4\cos 2t, -4\sin 2t)$

force $= -12(\cos 2t, \sin 2t)$.




force = centripetal force.

Like questions 13, 14, 23.

The acceleration, initial velocity and initial position of a particle are

a(t) = (1,2,3), v(0) = (2, -1, 1),  c(0) = (3,2,1) .

Find  c(t).

Solution: $v(t) = \int a(t)\,dt = (t, 2t, 3t)$ + constant

$v(0) = (2,-1,1)$ = constant

$v(t) = (t, 2t, 3t) + (2, -1, 1)$

Next $c(t) = \int v(t)\,dt$

$= \left(\frac{t^2}{2}, t^2, \frac{3t^2}{2}\right) + (2t, -t, t)$ + constant

$c(0)$ = constant $= (3,2,1)$

$c(t) = \left(3+2t+\frac{t^2}{2}, 2-t+t^2, 1+t+\frac{3t^2}{2}\right)$